PLANE PROBLEMS OF AEROTHERMOPTICS

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An exact solution is obtained of the refraction problem in a two-dimensionally inhomogeneous medium. Light propagation in nonisothermal gas streams is examined, and the influence of heat exchange on the light beam trajectory and intensity distribution is investigated.

In connection with the need to produce optical waveguides for optical communications systems, the production of thermogasdynamic waveguides is of great value [1-3]. Light propagation in inhomogeneously heated laminar subsonic ($M \leq 0.2$) gas streams, which it is proposed to use to control light beams, is considered in this paper.

1. Light propagation in a medium is described by a system of Maxwell equations (secondary charges and currents are assumed absent). The inhomogeneity of the dielectric permittivity field (for optical frequencies $\mu = 1$ [4]) is determined by convection which depends on absorption of radiation by the medium. Light beams in which the fields **E** and **H** are much less than the intermolecular fields, and moreover, whose radiation energy is much less than the internal gas energy $E^2 << 8\pi C_V T$ (C_V is the isochoric specific heat per unit volume in the system of units) are considered. For air at room temperature we should have $E << 3 \cdot 10^2$ W/cm or the radiation power <<1 GW/m². It is then sufficient to limit oneself to a linear relationship between **E** and **H** in the fixed gas and electrostriction and the electrocaloric effect can be neglected (which would result in excessive accuracy if taken into account since the gas is considered incompressible). The gas incompressibility also results in the absence of anisotropy ε because of the inhomogeneity of the gas velocity (the Maxwell dynamooptics effect). If the frequency dispersion ε of the gas at rest is negligible in the frequency band under consideration and there is no spatial dispersion, then the spatial dispersion ε in a moving medium which occurs because of field entrainment by the medium can be neglected. Therefore, the radiation does not influence convection, but convection influences the radiation because of the dependence of ε on the thermodynamic gas flow parameters.

To the accuracy of quantities on the order of vc^{-1}, the material equations in a moving medium have the form [4]

$$\mathbf{D} = \varepsilon \mathbf{E} + \frac{\varepsilon - 1}{c} [\mathbf{VH}], \tag{1}$$

$$\mathbf{B} = \mathbf{H} + \frac{\varepsilon - 1}{c} [\mathbf{EV}]. \tag{2}$$

In gases of nonpolar molecules (in dry air, for example) [5]

$$\varepsilon = 1 + 4\pi\alpha N. \tag{3}$$

Since $4\pi\alpha N << 1$ (for air $\varepsilon - 1 \approx 10^{-3}$), then to the accuracy of quantities of the order $4\pi N\alpha v c^{-1}$, we obtain $\mathbf{D} = \varepsilon \mathbf{H}$, $\mathbf{B} = \mathbf{H}$ from (1) and (2). To the same accuracy, the light phase velocity in a moving medium without dispersion can be considered invariant [4]. It is sufficient to consider the two-dimensional system of Maxwell equations for light rays being propagated along plane inhomogeneities. In first-order perturbation theory in $4\pi\alpha N$, the Cartesian **E** and **H** components satisfy the scalar wave equation

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = \frac{\varepsilon(x, y)}{c^2} \frac{\partial^2 U}{\partial t^2}. \tag{4}$$

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To determine the ray characteristics, it is sufficient to limit oneself to (4) without taking account of the vector nature of the fields E and H. The properties of the characteristics of (4) result in a variational problem for the Fermat functional [6]:

$$S = \int n ds \to \text{extr}, \quad n = \sqrt{\varepsilon}, \tag{5}$$

where $ds = \sqrt{(dx)^2 + (dy)^2}$ is the ray length element. The Euler equation for the functional (5) (the ray equation) has the form

$$\frac{d^2y}{dx^2} = \frac{1}{n}\left(\frac{\partial n}{\partial y} - \frac{dy}{dx} \cdot \frac{\partial n}{\partial x}\right)\left[1 + \left(\frac{dy}{dx}\right)^2\right] \tag{6}$$

or

$$\frac{d\varphi}{dx} = \frac{\partial \ln n}{\partial y} - \operatorname{tg}\varphi \frac{\partial \ln n}{\partial x}, \quad \frac{dy}{dx} = \operatorname{tg}[\varphi(x, y)]. \tag{7}$$

The angle of refraction depends on n, hence it follows from the first equation in (7) that

$$\frac{\partial\varphi}{\partial \ln n} = \operatorname{ctg}(\varphi + \gamma), \quad \frac{\partial n}{\partial x}\left(\frac{\partial n}{\partial y}\right)^{-1} = \operatorname{tg}[\gamma(x, y)]. \tag{8}$$

from which

$$\varphi = \arccos\left[\frac{n_e}{n}\cos(\varphi_e + \gamma_e)\right] - \gamma \tag{9}$$

(the subscript e refers to the initial point of ray incidence). Equations (8) and (9) can be rewritten in the form

$$\frac{\partial\varphi}{\partial \ln n} = -\operatorname{tg}(\varphi - \zeta), \quad \operatorname{ctg}\zeta = \operatorname{tg}\gamma \tag{10}$$

where $n = n(x)$

$$\varphi = \arcsin\left[\frac{n_e}{n}\sin(\varphi_e - \zeta_e)\right] + \zeta.$$

where $\varphi = \arcsin(n_e n^{-1}\sin\varphi_e)$ for $n = n(x)$ and $\varphi = \arccos(n_e n^{-1}\cos\varphi_e)$ for $n = n(y)$. Relationships (9) and (10) are meaningful only for $|n_e n^{-1}\cos(\varphi_e + \gamma_e)| \leqslant 1$ or $|n_e n^{-1}\sin(\varphi_e - \zeta_e)| \leqslant 1$; therefore, $n_e \leqslant n$. Hence, a ray is propagated toward increasing n. Let us note the strong dependence of the angle of refraction φ on the ratio of the components of the gradient of n. The dependence of φ on n is less essential since $4\pi\alpha N \ll 1$. In an ideal gas $p = NkT$ and because of (3) and (5)

$$n \cong 1 + \frac{rp}{T}, \quad r = 2\pi\alpha k^{-1}. \tag{11}$$

The relative change in density due to the change in pressure is small for a subsonic stream and for $M \ll 0.2$ it does not exceed 2% [7]. Therefore, the inhomogeneity in the field T is the governing factor of the inhomogeneity in n, meaning, the curving of the ray trajectory. Therefore,

$$n = 1 + \frac{b}{T}, \tag{12}$$

where $b = rp = \text{const}$ (for air under normal conditions $b = 9\cdot10^{-2}$ K). It is proposed to produce a field inhomogeneity because of the change in T as well as N in gas lenses. However, if there are such heat and mass transfer problems, it is possible to limit oneself to a consideration of one of the factors and just the temperature inhomogeneity of n will henceforth be considered.

The amplitude dependences of the light wave field are conveniently studied by substituting $U = W(x, y)\exp(i\omega t)$ into (4). Then

$$\Delta W + k_0^2 n^2 W = 0, \tag{13}$$

where $k_0 = \omega c^{-1}$. For large k_0, Eq. (13) allows an asymptotic expansion of the form [8]

$$W = \exp[ik_0 S(x, y)] \sum_{m=0}^{\infty} (ik_0)^{-m} A_m(x, y), \tag{14}$$

which, when substituted into (13), results in a system of equations for the eikonal S and the amplitudes A_m

$$(\nabla S)^2 = n^2,$$
$$2\nabla A_m \nabla S + A_m \Delta S = -\Delta A_{m-1}, \quad A_{-1} = 0. \tag{15}$$

It can be shown that

$$\frac{\partial S}{\partial x} = n\cos\varphi, \quad \frac{\partial S}{\partial y} = n\sin\varphi; \tag{16}$$

therefore, the first equation in (15) is satisfied identically, and equations for the transfer of amplitudes are obtained from (15) and (16)

$$2n\cos\varphi \frac{dA_m}{dx} + \operatorname{sc}\gamma\csc(\varphi+\gamma)\frac{\partial n}{\partial y} A_m = -\Delta A_{m-1}. \tag{17}$$

2. Let us examine a particular case. When

$$\frac{\partial\varphi}{\partial x} = \frac{\partial \ln n}{\partial y}, \quad \frac{\partial\varphi}{\partial y} = -\frac{\partial \ln n}{\partial x}, \tag{18}$$

the first equation in (7) is satisfied identically and integration of (6) reduces to solving the second equation in (7).

The relationships (18) are the Cauchy–Riemann conditions for an analytic function $\psi(z) = \varphi + i \ln n$, $z = x + iy$, therefore

$$\Delta\varphi = 0, \quad \Delta \ln n = 0. \tag{19}$$

If $N(x, y) = N_0 + \delta N(x, y)$, $N_0 = \text{const}$, $|\delta N| N^{-1} \ll 1$, and (3) and (5) are taken into account; to the accuracy of terms of the order of $\delta N\, N^{-1}$ inclusive, then the second equation in (19) will result in the diffusion equation

$$\Delta N = 0. \tag{20}$$

The validity of the converse assertion follows from the Taylor series expansion of ε in the neighborhood of the point N_0, $\varepsilon = \varepsilon_0 + (d\varepsilon/dN)\,\delta N + \ldots$. If $N = N(T)$, then the heat conduction is the cause of diffusion and under the constraints mentioned for N

$$\Delta T = 0. \tag{21}$$

The validity of the converse assertion is proved analogously. By giving the boundary conditions for N and T we determine n, meaning also φ. The lines $\varphi = \text{const}$, $n = \text{const}$ are orthogonal; hence the light ray will be propagated at a definite (constant) angle to the lines of constant concentration or isotherms (here the requirement for a two-dimensional inhomogeneity in n is absolute).

It should be noted that conditions (18) permit the determination of the exact solution of (13). Let us rewrite (13) along the characteristics, then

$$\frac{d^2W}{dS^2} n^2 + \Delta S \frac{dW}{dS} + k_0^2 n^2 W = 0,$$

which agrees with the equation of linearly damped oscillations. The second member governing wave scattering is missing since it is seen from (16) and (18) that $\Delta S = 0$. Then

$$W(S) = M_1 \exp(-ik_0 S) + M_2 \exp(ik_0 S),$$

where M_1 and M_2 are constants of integration. Here the WKB approximation agrees with the exact solution of the problem.

The problem considered corresponds to hydrostatic equilibrium, which is determined by the gas properties, the shape of the domain, and the boundary conditions [9]. For air between horizontal plates, between which the spacing is l, the temperature of the lower plate T_1 and the upper T_2, and the equilibrium condition is $Ra \lesssim 1.7 \cdot 10^3$ [10]. For $l = 0.01$ m, the temperature T_1 should not exceed 312°K.

The substantially lower gradients of n in a fixed inhomogeneously heated gas as compared with those possible in convective gas lenses make control of the ray trajectories relatively less efficient.

3. Let us examine light propagation in convective gas flows ($Pr \approx 1$). For the forced flow of a homogeneous stream with velocity u_∞ and temperature T_∞ near an isothermal plate heated to the temperature $T_w > T_\infty$ [7]

$$T = T_w - (T_w - T_\infty) f\left(\frac{y}{2}\sqrt{\frac{u_\infty}{\nu x}}\right). \tag{22}$$

It follows from (9) that

$$\frac{dy}{dx} \cong \varphi \cong -\gamma + \sqrt{2(n - n_e)\, n^{-1}}. \tag{23}$$

for a light ray being propagated near a plate with $y_e = 0$, $0 \leqslant \varphi_e \ll \pi/2$. Estimating the second term in the right-hand side of (23) as $\sqrt{2(n_\infty - n_w)}$ and taking account of (8) and (12), we obtain the trajectory equation ($y \ll x$)

$$y \cong 2\sqrt{2b(T_w - T_\infty)(T_w - T_\infty)^{-1}}\,(x - \sqrt{x_e x}). \tag{24}$$

If the concept of a finite thickness boundary layer $\delta(x) \backsim \sqrt{x}$ [7] is used, then curvature of the trajectory is approximately similar to "inversion" of $\delta(x)$. The angle of trajectory deviation in air is a quantity on the order of 0.2° in the laminar flow section to which $Re_x \lesssim 10^6$ corresponds for $T_w - T_\infty = 50$°K, $T_\infty = 300$°K and $u_\infty = 10$ m/sec. The insignificant deviation of the ray trajectory from a straight line in such flows indicates the legitimacy of the often used approximation of paraxial optics (small angle approximation) [1, 2]. The light beam amplitude for m = 0 as follows from (17) [7] will have the form

$$A_0 \cong A_{0e} \exp\{-b\, \mathrm{Nu}_x (T_w - T_\infty) T_w^{-2}\}, \tag{25}$$

$$\mathrm{Nu}_x = 0.332\sqrt{\mathrm{Re}_x}, \tag{26}$$

from which the influence of the heat transfer on the ray intensity is seen (compression of the ray tube).

The smallness of the ray deviation angles and therefore the smallness of the optical strengths of the gas lenses examined in [1, 2] do not permit the solution of the problem of turning a ray through an angle greater than tenths of a degree. Meanwhile, it is seen from (9) or (10) that greater changes in φ can be assured because of the abrupt change in γ or ζ, as is possible, e.g., in jet flows or wakes.

In the aerodynamic wave far behind the longitudinally streamlined heated isothermal plate of length $L (x > 3L)$ [11]

$$(u_\infty - u) \backsim x^{-0.5} \exp\left(-\frac{u_\infty y^2}{4\nu x}\right). \tag{27}$$

It can analogously be found that

$$T - T_\infty = \frac{C}{\sqrt{x}} \exp\left(-\frac{u_\infty y^2}{4\nu x}\right). \tag{28}$$

The enthalpy flux in the wake in the x direction is determined by the plate heat transfer

$$2\int_0^L q\,dx = \rho C_p u_\infty \int_{-\infty}^{+\infty} (T - T_\infty)\, dy,$$

where $q = 0.332\lambda(T_w - T_\infty)\sqrt{u_\infty(\nu x)^{-1}}$ [7]; therefore, $C = 0.664\,(T_w - T_\infty)\sqrt{L\pi^{-1}}$. In the paraxial optics approximation, the trajectory equation has the form

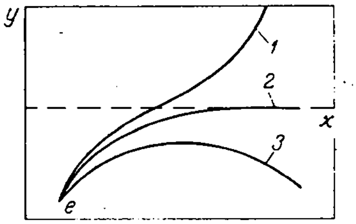

Fig. 1. Light ray trajectories in a heated aerodynamic wake or jet (x is the jet axis): 1) light ray passing through the jet; 2) light ray being propagated along the jet; 3) light ray reflected from the jet; e) initial point of ray incidence.

$$\frac{d^2y}{dx^2} = \frac{\partial n}{\partial y} = -\frac{b}{T_\infty} \cdot \frac{\partial T}{\partial y}.$$

Let us estimate the angles of trajectory deviation. Within the limits of a wake of thickness $\delta(x)$ ($\delta \simeq 2\sqrt{\nu x u_\infty^{-1}}$)

$$\frac{\partial T}{\partial y} \simeq -0.664\,(T_w - T_\infty) \sqrt{\frac{L}{\pi x}} \cdot \frac{u_\infty \delta}{2\nu x},$$

$$\frac{dy}{dx} \simeq 0.664\,\frac{b}{T_\infty} \sqrt{\mathrm{Re}_L}\,(T_w - T_\infty) \ln \frac{x}{x_e}, \quad (\varphi_e = 0).$$

In the wake behind a plate with $L = 0.15$ m, $x_e = 0.45$ m, $x = 1.2$ m, $\varphi \simeq 1°$. This indicates that cold jets are comparatively more efficient focusing systems than the gas lenses ordinarily used, for which the refraction angles are on the order of 0.2°.

From (9), (12), and (28) we can obtain

$$\varphi \cong \varphi_e + \gamma_e - \gamma,$$

$$\gamma \cong \operatorname{arctg}\left\{\frac{\nu}{u_\infty y}\left(1 - \frac{u_\infty y^2}{2\nu x}\right)\right\}, \quad u_\infty = 9\mathrm{Nu}_L^2 \frac{\nu}{L}.$$

The ray behavior for different φ_e is shown in the figure. If $\varphi_e > 0$, $(\varphi_e + \gamma_e) > 90°$, then the ray passes through the jet (curve 1). It is reflected from the jet (curve 3) for $(\varphi_e + \gamma_e) = 90°$ and behaves as described by curve 2 for $(\varphi_e + \gamma_e) = 90°$, whose position is unstable. All the rays passing through the heated or cooled jet result in a mirage (Fig.1).

Actually, the jet source always is finite in width and the jets will usually be turbulent, which will result in equalization of T and φ, as well as wavering of the light beam.

The results obtained can be carried over to the case of other two-dimensional and axisymmetric flows under the condition that the ray trajectory lies in the same plane as the axis of stream symmetry.

NOTATION

x, y, longitudinal and transverse coordinates; y(x), ray trajectory; **E**, **H**, **D**, **B**, electric and magnetic field intensities and inductions; **V**, stream velocity; u, velocity projection on the x axis; N, molecule concentration; T, temperature; p, pressure; ε, μ, dielectric and magnetic permittivities of the gas; α, molecule polarizability; c, speed of light in a vacuum, k_0, wave vector; ω, frequency; k, Boltzmann constant; ρ, density; ν, kinematic viscosity; M, Pr, Ra, Mach, Prandtl, and Rayleigh numbers; Re_x, Nu_x, local values of the Reynolds and Nusselt numbers.

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INFLUENCE OF ABSORPTION COEFFICIENT FLUCTUATIONS ON THE HEATING OF A WEAKLY ABSORBING MEDIUM BY INTENSE OPTICAL RADIATION

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The problem of heating the opacity fluctuations in a transparent solid medium by powerful optical radiation is numerically solved. The dependence of the absorption coefficient of the medium on the thermoelastic stresses is taken into account.

The need to solve heat conduction and thermoelasticity problems occurs every time in considering the question of destruction of transparent solid media by intense optical radiation. The role of the medium heating processes and the growth of thermoelastic stresses therein are appraised differently by different authors. In clarifying the reasons for the destruction of the optical glass used in lasers, the authors of [1] expressed the hypothesis that heating of the opaque microparticles in the glass and the subsequent growth of the thermoelastic stresses in the medium caused cracks which resulted in a loss of transparency. The idea of transparency inhomogeneities is used somewhat differently in [2-4], in which the importance of taking account of the temperature dependence of the absorption coefficient of the material is assumed. Destruction is caused by the temperature rise in the medium around foreign inclusions heated by the light and by the formation of an absorbing aureole around the impurity particle whose size and temperature grow avalanchelike for a sufficient radiation flux intensity because of nonlinearity of the problem. The role of the thermoelastic stresses, which can, in principle, also result in an increase in absorption by the medium because of narrowing of the forbidden band as the pressure grows, was not taken into account in [2-4]. Still another modification of taking account of the temperature dependences of the absorption coefficient of the medium is presented in [5], in which a medium with a relatively narrow forbidden band and a strong temperature dependence of the appropriate coefficients is examined. The medium here does not contain optical inhomogeneities. The distinction in the approaches listed above indicates the lack of complete clarity in the comprehension of the thermal processes causing the destruction of transparent solid materials by optical radiation and the need for further searches in this area.

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